University of California, Berkeley
Physics 110B Spring 2001 Section 1 (*Strovink*)

Problem Set 5

1. A particle with $\gamma\beta = 4/3$ decays into two massless particles with the same energy each.
(**a**) If the parent particle has mean proper life τ, calculate its mean flight path x before decay.
(**b**) Calculate the opening angle ψ between the two daughter particles.

2. Here's an adult version of Griffiths 12.35. In a pair annihilation experiment, a positron (mass m) with total energy $E = \gamma mc^2$ hits an electron (same mass, but opposite charge) at rest. (Griffiths has it the other way around, but that's unrealistic – it's easy to make a positron beam, but hard to make a positron target.) The two particles annihilate, producing two photons. (If only one photon were produced, energy-momentum conservation would force it to be a massive particle travelling at a velocity less than c.) If one of the photons emerges at angle θ relative to the incident positron direction, show that its energy E' is given by

$$\frac{mc^2}{E'} = 1 - \sqrt{\frac{\gamma - 1}{\gamma + 1}} \cos\theta \; .$$

(In particular, if the photon emerges perpendicular to the beam, its energy is equal to mc^2, independent of the beam energy. Similar results have been used to design clever experiments.)
[*Hint*: Griffiths 12.35 uses "convenient" values for γ and θ, but his solution to this problem is nevertheless full of messy algebra. Instead, as in class, write a four-vector equation expressing energy-momentum conservation, take the dot product of either side with itself, and get a concise result in a few lines.]

3. Griffiths 12.44.

4. Griffiths 12.45.

5. Griffiths 12.46.

6. A particle travelling with velocity $\beta c\hat{x}$ has a property represented by the contravariant four-vector h^μ. It is known that $p_\mu h^\mu = 0$, where p_μ is the particle's covariant four-momentum, where, by convention, repeated Greek indices are summed from 0 to 3. Write the components of h^μ in the laboratory as a function of those components in the particle's rest frame which are nonzero.

7. The metric tensor $g_{\mu\nu}$ is defined by

$$h_\mu \equiv g_{\mu\nu} h^\nu \; ,$$

where h^μ and h_μ are the contravariant and covariant versions of the four-vector h, whose invariant length^2 is equal to $h_\mu h^\mu$.
(**a**) Write out the elements of $g_{\mu\nu}$ (in flat spacetime, to which special relativity is pertinent).
(**b**) A contravariant four-tensor $T^{\mu\nu}$ is transformed to its covariant version $T_{\mu\nu}$ by two metric tensor multiplications:

$$T_{\mu\nu} \equiv g_{\mu\alpha} T^{\alpha\beta} g_{\beta\nu} \; .$$

Show that

$$g_{\mu\nu} = g^{\mu\nu} \; .$$

(**c**) Show that

$$g_{\mu\alpha} g^{\alpha\nu} = \delta_\mu^\nu \; ,$$

where the 4-dimensional Kronecker delta function satisfies $\delta_\mu^\nu = 0$ for $\mu \neq \nu$ and $\delta_\mu^\mu = 1$ for $0 \leq \mu \leq 3$.

8. Consider the antisymmetric contravariant tensor $H^{\mu\nu}$. Write out its covariant version $H_{\mu\nu}$ in matrix form, expressing each element of $H_{\mu\nu}$ in terms of the elements of $H^{\mu\nu}$.